2.53 A rod consisting of two cylindrical portions $AB$ and $BC$ is restrained at both ends. Portion $AB$ is made of steel ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}{}^\circ$F) and portion $BC$ is made of brass ($E_b = 15 \times 10^6$ psi, $\alpha_b = 10.4 \times 10^{-6}{}^\circ$F). Knowing that the rod is initially unstressed, determine (a) the normal stresses induced in portions $AB$ and $BC$ by a temperature rise of 65°F, (b) the corresponding deflection of point $B$.

2.56 For the rod of Prob. 2.53, determine the maximum allowable temperature change if the stress in the steel portion $AB$ is not to exceed 18 ksi and if the stress in the brass portion $CB$ is not to exceed 7 ksi.

**SOLUTION**

Allowable force in each portion

$AB: \sigma_{AB} = -18 \times 10^3$ psi, $A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4} (1.25)^2 = 1.2272$ in$^2$

$P = \sigma_{AB} A_{AB} = (-18 \times 10^3)(1.2272) = -22.090 \times 10^3$ lb.

$BC: \sigma_{BC} = -7 \times 10^3$ psi, $A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4} (2.25)^2 = 3.9761$ in$^2$

$P = \sigma_{BC} A_{BC} = (-7 \times 10^3)(3.9761) = -27.833 \times 10^3$ lb.

Smaller absolute value governs: $P = -22.090 \times 10^3$ lb.

Deformation due to P

$\delta_p = \frac{PL_{AB}}{E_s A_{AB}} + \frac{PL_{BC}}{E_b A_{BC}} = \frac{(22.090 \times 10^3)(12)}{(29 \times 10^6)(1.2272)} - \frac{(22.090 \times 10^3)(15)}{(15 \times 10^6)(3.9761)}$

$= -13.004 \times 10^{-3}$ in

Free thermal expansion

$\delta_T = L_{AB} \alpha_s (\Delta T) + L_{BC} \alpha_b (\Delta T) = (12)(6.5 \times 10^{-6}) (\Delta T) + (15)(10.4 \times 10^{-6}) (\Delta T)$

$= (234 \times 10^{-6}) (\Delta T)$

Total deformation is zero

$\delta_T + \delta_p = (234 \times 10^{-6}) (\Delta T) - 13.004 \times 10^{-3} = 0$

$\Delta T = 55.6 {}^\circ$ F