Implementation of the Wave Variable Method on a PHANTOM Omni Haptic Device

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ABSTRACT
Time delay is a serious problem for bilateral teleoperation systems. Even a small time delay in a bilateral teleoperation system will generally degrade the system’s performance and cause instability. An important approach that guarantees stability for any fixed time delay is the wave variable method. In this paper we present some recent material dealing with teleoperation systems using wave variables. In particular, we describe a wave variable scheme based on a family of scaling matrices for multiple degree-of-freedom bilateral teleoperation system. We include a derivation of a larger family of scaling matrices that will guarantee the system remains stable for a fixed time delay. A multiple degree-of-freedom bilateral teleoperation system using the new wave variable method is simulated using a SIMULINK model, and then is implemented on the PHANTOM Omni haptic device.

Keywords
Teleoperation, haptics, time delay, wave variables.

1. INTRODUCTION
Teleoperation systems are frequently used for a number of different tasks, e.g., handling toxic or harmful materials, dealing with remote environments such as underwater or space, and tasks that require extreme precision. However, one major problem that can be found in bilateral teleoperation systems is time delay. When the master and slave are located at a far distance from each other, the time delay is no longer negligible. In the late 1980’s Anderson and Spong [3] found that it is possible to stabilize a force reflecting teleoperation system that has a time delay, by exploiting scattering theory. Later, Niemeyer and Slotine [7] presented the wave variable method in a more intuitive, physically motivated formalism based on passivity [8]. More recently, Munir and Book [6] included predictive techniques in the wave variable method to handle the time-varying delays encountered on the internet.

In the wave variable method, wave variables are used in place of the more conventional power variables like velocity and force. It was found that when forces and velocities were transformed into wave variables and transmitted at both the master and slave sides, systems could remain stable even with some time delay. In spite of these recent advances, issues regarding stability and performance of systems with variable time delay still remain a challenge that must be addressed if teleoperation is to reach its full potential.

In this article, we examine a generalization of the wave variable method to multiple degree-of-freedom teleoperators [4]. We more carefully examine the derivation of the wave parameters, and extend the choice of those parameters. A brief discussion of the wave variable method is presented in the next section along with descriptions of two generalizations to multiple degree-of-freedom systems. A clarification and extension of this generalization is given in Section 3. Section 4 contains a simulation of a 3 DOF teleoperator to illustrate the effect that the new parameters have on the wave variables. Section 5 explains the implementation using the PHANTOM Omni, and also shows the results of the experiments. Conclusions appear in Section 6.

2. THE WAVE VARIABLE METHOD
In a basic bilateral setup, the master location sends information to the slave location while at the same time the slave is sending back different information. In our case the feedback in the system is in the form of a force that the user will feel. As long as no time delay is present this system performs well, i.e., the slave’s behavior tracks the master’s behavior, both with force and velocity. Without any form of control present, once a small amount of delay is introduced into the system, the performance will quickly degrade, and the system may even become unstable. Figure 1 is an illustration of a haptic bilateral teleoperation system with wave variable transformations present. The parameter $T$ represents the time delay between the master and slave.

Figure 1: Bilateral teleoperation system with wave transformations.

The wave transformation relations for the single degree-of-freedom case are given by

$$\dot{u}_m(t) = u_m(t - T)$$
$$\dot{v}_m(t) = v_m(t - T).$$

The wave transformations for the left wave junction are given by

$$G_{L}(s)$$

$$G_{S}(s)$$

$G_{L}(s)$

$G_{S}(s)$

$u_m$

$u_s$

$\dot{u}_m$

$\dot{u}_s$

$\dot{v}_m$

$\dot{v}_s$

$T$

$T$

$\dot{v}_d$

$\dot{v}_d$

$G_{L}(s)$

$G_{S}(s)$

$u_m$

$u_s$

$\dot{u}_m$

$\dot{u}_s$

$\dot{v}_m$

$\dot{v}_s$

$T$

$T$

$\dot{v}_d$

$\dot{v}_d$
\[ u_w(t) = \frac{b\dot{\theta}_m(t) + \tau_m(t)}{\sqrt{2b}} \tag{2} \]
\[ v_w(t) = \frac{b\dot{\theta}_m(t) - \tau_m(t)}{\sqrt{2b}} \]

and that for the right wave junction are given by
\[ u_s(t) = \frac{b\dot{\theta}_w(t) + \tau_{pd}(t)}{\sqrt{2b}} \tag{3} \]
\[ v_s(t) = \frac{b\dot{\theta}_w(t) - \tau_{pd}(t)}{\sqrt{2b}} \]

Although the strictly positive parameter \( b \) can be chosen arbitrarily, it defines a characteristic impedance associated with the wave variables and directly affects the system behavior [7].

Equations (2) and (3) are for 1 DOF systems. To implement the wave variable method on a system that has more than one degree of freedom, the equations for the transforms must be generalized. Niemeyer and Slotine [7] suggest making

\[ \dot{\theta}_m(t) = A_w \dot{\theta}_m(t) + B_w \tau_m(t) \tag{4} \]
\[ \dot{\theta}_w(t) = C_w \dot{\theta}_w(t) - D_w \tau_{pd}(t) \]

and

\[ \dot{\theta}_m(t) = A_s \dot{\theta}_m(t) + B_s \tau_m(t) \tag{5} \]
\[ \dot{\theta}_w(t) = C_s \dot{\theta}_w(t) - D_s \tau_{pd}(t) \]

where \( A_w, B_w, C_w, \) and \( D_w \) are \( n \times n \) scaling matrices and \( n \) is the number of degrees of freedom of the teleoperation system. These matrices cannot be chosen arbitrarily; it is necessary to determine conditions for the scaling matrices to guarantee passivity [8]. To accomplish this we will define the power-flow at each side to be

\[ \dot{\theta}_m^T \tau_m = \frac{1}{2} u_m^T u_m - \frac{1}{2} v_m^T v_m \tag{6} \]

for the master side and

\[ \dot{\theta}_w^T \tau_{pd} = -\frac{1}{2} u_w^T u_w + \frac{1}{2} v_w^T v_w \tag{7} \]

for the slave side. Substituting equations (4) and (5) into equation (6) or (7), expanding, and matching matrix coefficients yields the requirements

\[ A_w^T A_w = C_w^T C_w \]
\[ B_w^T B_w = D_w^T D_w \tag{8} \]

and also that

\[ I = \frac{1}{2} (2 A_w^T B_w + 2 C_w^T D_w) \tag{9} \]

Munir and Book [4]-[6] derive conditions on \( A_w \) and \( B_w \) to ensure that (8) and (9) are satisfied. In particular, they note that the scaling matrices must be nonsingular and consider the special case

\[ A_w = C_w \]
\[ B_w = D_w \tag{10} \]

so that equation (8) is satisfied. Using this relationship, equation (9) reduces to

\[ I = 2 A_w^T B_w. \tag{11} \]

Munir and Book then restrict \( A_w \) to be symmetric but not necessarily positive definite and prove that the resulting family of scaling matrices results in a stable system by showing that passivity is ensured using the norm of the scattering matrix.

Although this is more general than what was proposed in [7], because of the specific choices made, only a restricted class of scaling matrix is determined. In this article, we will extend the family of scaling matrices that result in stability and discuss the significance of this extension in terms of the wave variables themselves.

### 3. DERIVATION OF A LARGER FAMILY OF SCALING MATRICES

To extend the family of scaling matrices originally proposed by Munir and Book, we will first derive the whole family of matrices satisfying (8) and (9). From equation (8) it is clear that \( A_w^T \) and \( C_w^T \) have the same column space and that \( B_w \) and \( D_w \) have the same row space. This observation along with equation (9) implies that all four scaling matrices must be nonsingular. Since all square root decompositions of nonsingular square matrices such as those given in (8) are related by pre-multiplication by an orthogonal matrix, it follows that

\[ C_w = Q_1 A_w \]
\[ D_w = Q_2 B_w \tag{12} \]

where \( Q_1 \) and \( Q_2 \) are \( n \times n \) orthogonal matrices. Now this is not enough to guarantee that (9) is satisfied. We will say that \( A_w \) and \( B_w \) are compatible with respect to (8) and (9) if there are orthogonal matrices \( Q_1 \) and \( Q_2 \) so that \( A_w, B_w, C_w = Q_1 A_w, \) and \( D_w = Q_2 B_w \) satisfy (8) and (9). Substituting (12) into (9) and performing the required manipulations gives the following necessary and sufficient condition for \( A_w \) and \( B_w \) to be compatible in this sense:

\[ A_w B_w^T + B_w A_w^T = I. \tag{13} \]

With some algebraic manipulation [1], it can then be shown that the characterizing condition for \( A_w, B_w, C_w, \) and \( D_w \) to satisfy (8) and (9) is that the following hold:

1. \( A_w \) is nonsingular.
2. \( B_w = \frac{1}{2} A_w^{-T}. \)
3. \( C_w = Q A_w \) where \( Q \) is any \( n \times n \) orthogonal matrix.
4. \( D_w = \frac{1}{2} QA_w^{-T}. \)

Note that setting \( A_w = \sqrt{\frac{b}{2}} \) and \( Q = 1 \) for the scalar case results in exactly the same solution as (2) and (3).

Since choosing a set of scaling matrices requires the selection of an \( n \times n \) nonsingular matrix \( A_w \) and an \( n \times n \) orthogonal matrix \( Q \), there are a total of \( n^2 + n(n-1)/2 = 3n^2/2 - n/2 \) degrees of freedom in choosing the scaling matrices. However,
because the collection of scaling matrices presented in [4]-[6] are uniquely defined by the selection of a single symmetric matrix $A_w$, the family of scaling matrices in [4]-[6] is only $n(n+1)/2$-dimensional.

It is then natural to ask how this extension of the scaling matrices affects the wave variables. To see this, we first consider the effect of $Q$. The $v$ wave variables are given in equations (4) and (5). Substituting in the expressions for $C_w$ and $D_w$ yields

$$v_m(t) = QA_w \dot{\theta}_m(t) - \frac{1}{2} QA_w^T \tau_m(t)$$

which clearly demonstrates that $Q$ merely applies an orthogonal transformation to the $v$-variable, i.e., it will merely rotate and/or reflect the $v$-variable. The same holds for $v_i(t)$. This will clearly have no effect on the power flow equations (6) and (7).

Next, consider the effect of allowing the matrix $A_w$ to be nonsymmetric. There is a well-known result in matrix theory called the polar decomposition that states that any square matrix $M$ can be written as a product $M = PU^T$ where $P$ is a symmetric positive definite matrix and $U$ is an orthogonal matrix. Setting $M = A_w^c$ gives that $A_w = UP_w$ for some suitable orthogonal matrix $U$ and symmetric positive definite matrix $P_w$. The $u$ wave variable then becomes

$$u_m(t) = QP_w \dot{\theta}_m(t) + \frac{1}{2} (QP_w)^T \tau_m(t)$$

so that the $u$ -variable is merely rotated and/or reflected. A similar statement holds for the $v$-variable.

4. SIMULATION RESULTS

In order to test our results, a bilateral teleoperation system was set up using both Matlab and SIMULINK. This allowed us test multiple different master and slave models, as well as easily vary both the time delay amount and the wave variables used. For this paper the performance of the wave variable method was tested using a 3 degree-of-freedom linear model for simplicity, a more difficult example is shown in the next section. The equations of motion for both the master and slave manipulators were given by

$$\tau_m = J_m \dot{\theta} + B_m \dot{\theta}$$

$$\tau_{pd} = J_s \dot{\theta} + B_s \dot{\theta}$$

where $\tau_m$, $\tau_{pd}$ are the input torques, $J_m$, $J_s$ are the desired $3 \times 3$ constant symmetric positive definite inertia matrices, and $B_m$, $B_s$ are the damping matrices with the same qualities as $J_m$, $J_s$. A simple PI-controller was used on the slave side to generate the force necessary to move the slave. The input to the controller was the difference between the delayed, desired velocity, and the actual slave velocity. With the wave variable parameters and the equations of motion necessary to complete the system, we can simulate a 3 degree-of-freedom bilateral teleoperation system.

The simulated system used the following master and slave model:

$$J_s = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}, B_s = \begin{bmatrix} 8 & 2 & 2 \\ 2 & 8 & 2 \end{bmatrix}.$$  

Because the model is symmetric, a simple, symmetric set of wave variables were chosen. As can be seen in Figure 2, the behavior of the linear 3 degree-of-freedom slave is similar to the behavior of the master. In this simulation, the input torques were 7, 3, and -4 N-m, respectively. The communication line was set up to have a 0.5 sec time delay for both the master to slave, as well as the slave to master data transfer.

Figure 2: The response of a 3-DOF system with a $2T = 1$ sec total time delay.

Figure 3: Difference in torque, position, and velocity for a 3-DOF system with a $2T = 1$ sec total time delay.
Figure 2 represents the simulation results for a given fixed $A_w$, where $Q$ is allowed to be any orthogonal matrix. Figure 3 shows the differences in torque, position, and velocity for the same system. As can be seen, the slave manipulator was able to match the performance of the master manipulator with respect to torque, position, and velocity. Due to the nature of the wave variable method, it can be shown that the system will stabilize for any amount of constant time delay. This was tested by using many different amounts of time delay and changing the wave variables accordingly to improve performance. Also, no matter which $Q$ is used, the overall system output will not change. What do change are the wave variables that are sent across the communication line [2]. Because $Q$ is a rotation matrix, when it is any value other then the zero matrix, the wave variable $v_s$ will be rotated by the amount associated with the $Q$ matrix. Finally, when looking at $P_w = U^T A_w$, which is a rotated version of $A_w$, the same statement from before can be made again. This time however both $u_m$ and $v_s$ will be rotated, and again there will be no change to the system output.

5. PHANTOM OMNI IMPLEMENTATION

In order to implement the wave variable method into a bilateral teleoperation system using the PHANTOM Omni haptic device, we used several SIMULINK toolboxes. First, we used a toolbox from Handshake Prosense. The blocks in this toolbox allowed us to import the data from the Omni into SIMULINK, as well as feed back force data to the Omni for the user to feel. Also, the toolbox provided objects that could be placed in a virtual environment. These objects would generate a feedback force if contacted. We used the virtual reality toolbox so that we could view the slave along with the objects created in the virtual environment. The slave model that was used for experimentation was the same 3 degree-of-freedom linear model that was used in the prior simulations. Also, we again used a PI-controller in order to generate the desired force for the slave. Figure 4 shows the response of the PHANTOM Omni teleoperation system with a $2T = 1$ sec time delay. The master robot drew out a circular path around the virtual object and as can be seen in Figure 4, the slave tracks the master well. The slave follows a similar path to that of the master only delayed slightly in time. Figure 5 shows the differences in torque, position, and velocity for the PHANTOM system. Just like in the simulations, $Q$ was allowed to vary, and again the overall output was not affected.

6. CONCLUSIONS

In summary, we have presented the derivation of an extension of the wave variable method to multiple degree-of-freedom systems. We have shown that the collection of scaling matrices determined in [4]-[6] to preserve passivity is not complete and have determined a larger family of feasible scaling matrices. We have also shown how the new scaling matrices relate to those proposed in [4]-[6]. We have included simulations that implement the wave variable method into a bilateral teleoperation system with time delay using SIMULINK, as well as one using the PHANTOM Omni haptic device.

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8. REFERENCES


