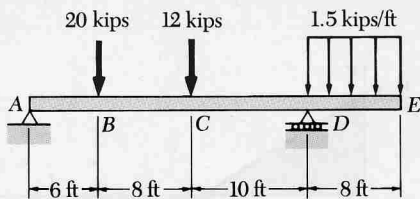


SAMPLE PROBLEM 7.3

Draw the shear and bending-moment diagrams for the beam and loading shown.



Solution. Considering the entire beam as a free body, we determine the reactions:

$$+\uparrow \Sigma M_A = 0:$$

$$D(24 \text{ ft}) - (20 \text{ kips})(6 \text{ ft}) - (12 \text{ kips})(14 \text{ ft}) - (12 \text{ kips})(28 \text{ ft}) = 0$$

$$D = +26 \text{ kips} \quad \mathbf{D} = 26 \text{ kips} \uparrow$$

$$+\uparrow \Sigma F_y = 0:$$

$$A_y - 20 \text{ kips} - 12 \text{ kips} + 26 \text{ kips} - 12 \text{ kips} = 0$$

$$A_y = +18 \text{ kips} \quad \mathbf{A}_y = 18 \text{ kips} \uparrow$$

$$\rightarrow \Sigma F_x = 0:$$

$$A_x = 0 \quad \mathbf{A}_x = 0$$

We also note that at both A and E the bending moment is zero; thus two points (indicated by dots) are obtained on the bending-moment diagram.

Shear Diagram. Since $dV/dx = -w$, we find that between concentrated loads and reactions the slope of the shear diagram is zero (i.e., the shear is constant). The shear at any point is determined by dividing the beam into two parts and considering either part as a free body. For example, using the portion of beam to the left of section I, we obtain the shear between B and C:

$$+\uparrow \Sigma F_y = 0: \quad +18 \text{ kips} - 20 \text{ kips} - V = 0 \quad V = -2 \text{ kips}$$

We also find that the shear is +12 kips just to the right of D and zero at end E. Since the slope $dV/dx = -w$ is constant between D and E, the shear diagram between these two points is a straight line.

Bending-Moment Diagram. We recall that the area under the shear curve between two points is equal to the change in bending moment between the same two points. For convenience, the area of each portion of the shear diagram is computed and is indicated in parentheses on the diagram. Since the bending moment M_A at the left end is known to be zero, we write

$$\begin{aligned} M_B - M_A &= +108 & M_B &= +108 \text{ kip} \cdot \text{ft} \\ M_C - M_B &= -16 & M_C &= +92 \text{ kip} \cdot \text{ft} \\ M_D - M_C &= -140 & M_D &= -48 \text{ kip} \cdot \text{ft} \\ M_E - M_D &= +48 & M_E &= 0 \end{aligned}$$

Since M_E is known to be zero, a check of the computations is obtained.

Between the concentrated loads and reactions the shear is constant; thus, the slope dM/dx is constant and the bending-moment diagram is drawn by connecting the known points with straight lines. Between D and E where the shear diagram is an oblique straight line, the bending-moment diagram is a parabola.

From the V and M diagrams we note that $V_{\max} = 18 \text{ kips}$ and $M_{\max} = 108 \text{ kip} \cdot \text{ft}$.

