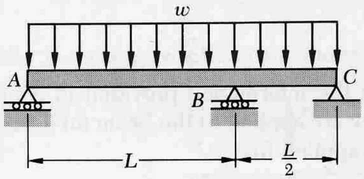
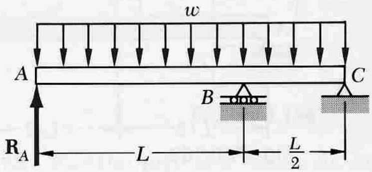
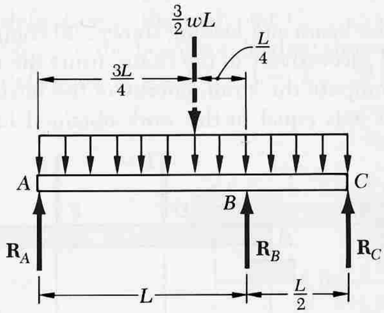
SAMPLE PROBLEM 10.7



For the uniform beam and loading shown, determine the reactions at the supports.





Castigliano's Theorem. The beam is indeterminate to the first degree and we choose the reaction $\mathbf{R}_A$ as redundant. Using Castigliano's theorem, we shall determine the deflection at A due to the combined action of $\mathbf{R}_A$ and the distributed load. Since the flexural rigidity EI is constant, we write

$$y_A = \int \frac{M}{EI}\left(\frac{\partial M}{\partial R_A}\right) dx = \frac{1}{EI}\int M \frac{\partial M}{\partial R_A} dx \qquad (1)$$

The integration will be performed separately for portions AB and BC of the beam. Finally, $\mathbf{R}_A$ is obtained by setting y_A equal to zero.

Free Body: Entire Beam. We express the reactions at B and C in terms of R_A and the distributed load

$$R_B = \tfrac{9}{4}wL - 3R_A \qquad R_C = 2R_A - \tfrac{3}{4}wL \qquad (2)$$

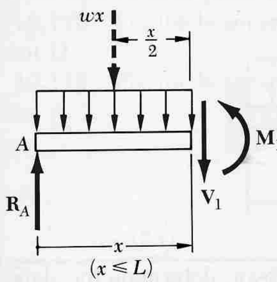
Portion AB of Beam. Using the free-body diagram shown, we find the bending moment M_1 and compute its partial derivative with respect to R_A.

$$M_1 = R_A x - \frac{wx^2}{2} \qquad \frac{\partial M_1}{\partial R_A} = x$$

Substituting into Eq. (1) and integrating from A to B, we have

$$\frac{1}{EI}\int M_1 \frac{\partial M}{\partial R_A} dx = \frac{1}{EI}\int_0^L \left(R_A x^2 - \frac{wx^3}{2}\right) dx = \frac{1}{EI}\left(\frac{R_A L^3}{3} - \frac{wL^4}{8}\right) \qquad (3)$$

From A to B



Portion BC of Beam. We have

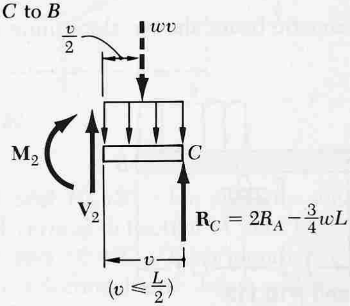
$$M_2 = \left(2R_A - \frac{3}{4}wL\right)v - \frac{wv^2}{2} \qquad \frac{\partial M_2}{\partial R_A} = 2v$$

Substituting into Eq. (1) and integrating from C, where $v = 0$, to B, where $v = \frac{1}{2}L$, we have

$$\frac{1}{EI}\int M_2 \frac{\partial M_2}{\partial R_A} dv = \frac{1}{EI}\int_0^{L/2} \left(4R_A v^2 - \frac{3}{2}wLv^2 - wv^3\right) dv$$

$$= \frac{1}{EI}\left(\frac{R_A L^3}{6} - \frac{wL^4}{16} - \frac{wL^4}{64}\right) = \frac{1}{EI}\left(\frac{R_A L^3}{6} - \frac{5wL^4}{64}\right) \qquad (4)$$

From C to B



Reaction at A. Adding the expressions obtained in (3) and (4), we determine y_A and set it equal to zero

$$y_A = \frac{1}{EI}\left(\frac{R_A L^3}{3} - \frac{wL^4}{8}\right) + \frac{1}{EI}\left(\frac{R_A L^3}{6} - \frac{5wL^4}{64}\right) = 0$$

Solving for R_A, $\qquad R_A = \frac{13}{32}wL \qquad\qquad R_A = \frac{13}{32}wL \uparrow$ ◀

Reactions at B and C. Substituting for R_A into Eqs. (2), we obtain

$$R_B = \frac{33}{32}wL \uparrow \qquad R_C = \frac{wL}{16} \uparrow$$ ◀