

Example 9.05

Determine the maximum deflection for the beam of Example 9.04.

Determination of Point K Where Slope Is Zero. We recall from Example 9.04 that the slope at point D , where the load is applied, is negative. It follows that point K , where the slope is zero, is located between D and the support B (Fig. 9.28). Our computations, therefore, will be simplified if we relate the slope at K to the slope at B , rather than to the slope at A .

Since the slope at A has already been determined in Example 9.04, the slope at B may be obtained by writing

$$\theta_B = \theta_A + \theta_{B/A} = \theta_A + A_1 + A_2$$

$$\theta_B = -\frac{7PL^2}{128EI} + \frac{3PL^2}{128EI} + \frac{9PL^2}{128EI} = \frac{5PL^2}{128EI}$$

Observing that the bending moment at a distance u from end B is $M = \frac{1}{4}Pu$ (Fig. 9.29a), we express the area A' located between K and B under the (M/EI) diagram (Fig. 9.29b) as

$$A' = \frac{1}{2} \frac{Pu}{4EI} u = \frac{Pu^2}{8EI}$$

By the first moment-area theorem, we have

$$\theta_{B/K} = \theta_B - \theta_K = A'$$

and, since $\theta_K = 0$,

$$\theta_B = A'$$

Substituting the values obtained for θ_B and A' , we write

$$\frac{5PL^2}{128EI} = \frac{Pu^2}{8EI}$$

and, solving for u ,

$$u = \frac{\sqrt{5}}{4} L = 0.559L$$

Thus, the distance from the support A to point K is

$$AK = L - 0.559L = 0.441L$$

Maximum Deflection. The maximum deflection $|y|_{\max}$ is equal to the tangential deviation $t_{B/K}$ and, thus, to the first moment of the area A' about a vertical axis through B (Fig. 9.29b). We write

$$|y|_{\max} = t_{B/K} = A' \left(\frac{2u}{3} \right) = \frac{Pu^2}{8EI} \left(\frac{2u}{3} \right) = \frac{Pu^3}{12EI}$$

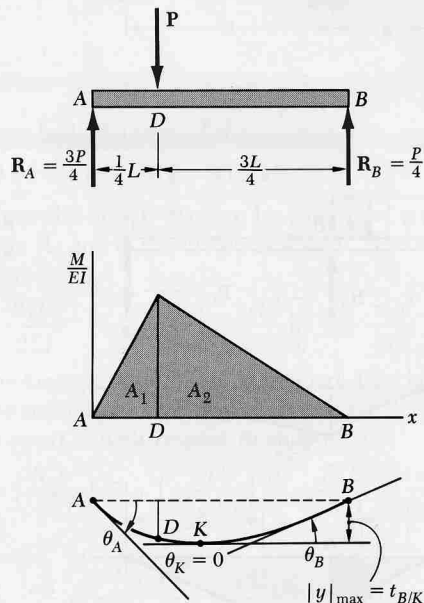


Fig. 9.28

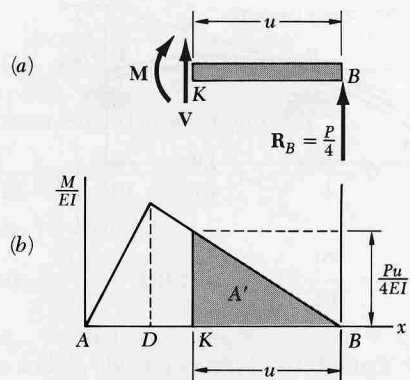


Fig. 9.29

Substituting the value obtained for u , we have

$$|y|_{\max} = \frac{P}{12EI} \left(\frac{\sqrt{5}}{4} L \right)^3 = 0.01456PL^3/EI$$