Fatigue Failure Theories under Static Loading

PROBLEMS

ANALYSIS

6–1 A ductile hot-rolled steel bar has a minimum yield strength in tension and compression of 50 kpsi. Using the distortion-energy and maximum-shear-stress theories determine the factors of safety for the following plane stress states:
(*a*) $\sigma_x = 12$ kpsi, $\sigma_y = 6$ kpsi
(*b*) $\sigma_x = 12$ kpsi, $\tau_{xy} = -8$ kpsi
(*c*) $\sigma_x = -6$ kpsi, $\sigma_y = -10$ kpsi, $\tau_{xy} = -5$ kpsi
(*d*) $\sigma_x = 12$ kpsi, $\sigma_y = 4$ kpsi, $\tau_{xy} = 1$ kpsi

ANALYSIS

6–2 Repeat Prob. 6–1 for:
(*a*) $\sigma_A = 12$ kpsi, $\sigma_B = 12$ kpsi
(*b*) $\sigma_A = 12$ kpsi, $\sigma_B = 6$ kpsi
(*c*) $\sigma_A = 12$ kpsi, $\sigma_B = -12$ kpsi
(*d*) $\sigma_A = -6$ kpsi, $\sigma_B = -12$ kpsi

ANALYSIS

6–3 Repeat Prob. 6–1 for a bar of AISI 1020 cold-drawn steel and:
(*a*) $\sigma_x = 180$ MPa, $\sigma_y = 100$ MPa
(*b*) $\sigma_x = 180$ MPa, $\tau_{xy} = 100$ MPa
(*c*) $\sigma_x = -160$ MPa, $\tau_{xy} = 100$ MPa
(*d*) $\tau_{xy} = 150$ MPa

ANALYSIS

6–4 Repeat Prob. 6–1 for a bar of AISI 1018 hot-rolled steel and:
(*a*) $\sigma_A = 100$ MPa, $\sigma_B = 80$ MPa
(*b*) $\sigma_A = 100$ MPa, $\sigma_B = 10$ MPa
(*c*) $\sigma_A = 100$ MPa, $\sigma_B = -80$ MPa
(*d*) $\sigma_A = -80$ MPa, $\sigma_B = -100$ MPa

ANALYSIS

6–5 Repeat Prob. 6–3 by first plotting the failure loci in the σ_A, σ_B plane to scale; then, for each stress state, plot the load line and by graphical measurement estimate the factors of safety.

ANALYSIS

6–6 Repeat Prob. 6–4 by first plotting the failure loci in the σ_A, σ_B plane to scale; then, for each stress state, plot the load line and by graphical measurement estimate the factors of safety.

ANALYSIS

6–7 An ASTM cast iron has minimum ultimate strengths of 30 kpsi in tension and 100 kpsi in compression. Find the factors of safety using the MNS, BCM, M1M, and M2M theories for each of the following stress states. Plot the failure diagrams in the σ_A, σ_B plane to scale and locate the coordinates of each stress state.
(*a*) $\sigma_x = 20$ kpsi, $\sigma_y = 6$ kpsi
(*b*) $\sigma_x = 12$ kpsi, $\tau_{xy} = -8$ kpsi
(*c*) $\sigma_x = -6$ kpsi, $\sigma_y = -10$ kpsi, $\tau_{xy} = -5$ kpsi
(*d*) $\sigma_x = -12$ kpsi, $\tau_{xy} = 8$ kpsi

ANALYSIS

6–8 For Prob. 6–7, case (*d*), estimate the factors of safety from the four theories by graphical measurements of the load line.

DECISIONS IN DESIGN

ANALYSIS

6–9 Among the decisions a designer must make is selection of the failure locus that is applicable to the material and its static loading. A 1020 hot-rolled steel has the following properties: $S_y = 42$ kpsi, $S_{ut} = 66.2$ kpsi, and true strain at fracture $\varepsilon_f = 0.90$. Plot the failure locus and, for the static stress states at the critical locations listed below, plot the load line and estimate the factor of safety analytically and graphically.
(*a*) $\sigma_x = 9$ kpsi, $\sigma_y = -5$ kpsi.
(*b*) $\sigma_x = 12$ kpsi, $\tau_{xy} = 3$ kpsi ccw.
(*c*) $\sigma_x = -4$ kpsi, $\sigma_y = -9$ kpsi, $\tau_{xy} = 5$ kpsi cw.
(*d*) $\sigma_x = 11$ kpsi, $\sigma_y = 4$ kpsi, $\tau_{xy} = 1$ kpsi cw.

DECISIONS IN DESIGN **6–10** A 4142 steel Q&T at 80°F exhibits $S_{yt} = 235$ kpsi, $S_{yc} = 275$ kpsi, and $\varepsilon_f = 0.06$. Choose and plot the failure locus and, for the static stresses at the critical locations, which are 10 times those in Prob. 6–9, plot the load lines and estimate the factors of safety analytically and graphically.

ANALYSIS **6–11** For grade 20 cast iron, Table A–24 gives $S_{ut} = 22$ kpsi, $S_{uc} = 83$ kpsi. Choose and plot the failure locus and, for the static loadings inducing the stresses at the critical locations of Prob. 6–9, plot the load lines and estimate the factors of safety analytically and graphically.

DECISIONS IN DESIGN **6–12** A cast aluminum 195-T6 has an ultimate strength in tension of $S_{ut} = 36$ kpsi and ultimate strength in compression of $S_{uc} = 35$ kpsi, and it exhibits a true strain at fracture $\varepsilon_f = 0.045$. Choose and plot the failure locus and, for the static loading inducing the stresses at the critical locations of Prob. 6–9, plot the load lines and estimate the factors of safety analytically and graphically.

ANALYSIS **6–13** An ASTM cast iron, grade 30 (see Table A–24), carries static loading resulting in the stress state listed below at the critical locations. Choose the appropriate failure locus, plot it and the load lines, and estimate the factors of safety analytically and graphically.

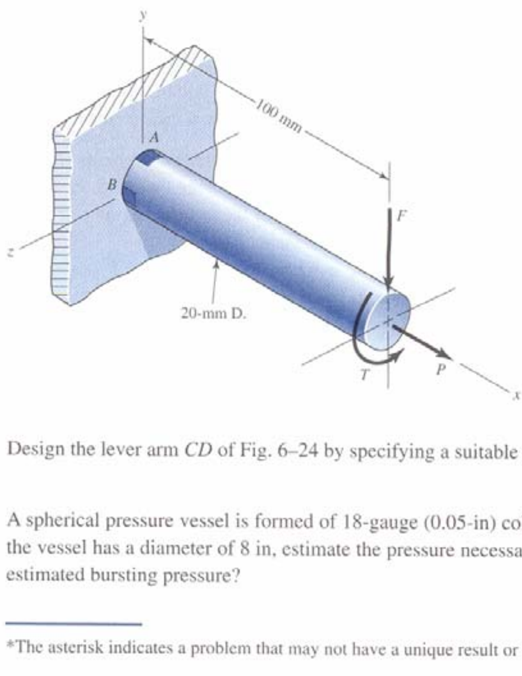
(*a*) $\sigma_A = 20$ kpsi, $\sigma_B = 20$ kpsi.

(*b*) $\tau_{xy} = 15$ kpsi.

(*c*) $\sigma_A = \sigma_B = -80$ kpsi.

(*d*) $\sigma_A = 15$ kpsi, $\sigma_B = -25$ kpsi.

ANALYSIS **6–14** This problem illustrates that the factor of safety for a machine element depends on the particular point selected for analysis. Here you are to compute factors of safety, based upon the distortion-energy theory, for stress elements at A and B of the member shown in the figure. This bar is made of AISI 1006 cold-drawn steel and is loaded by the forces $F = 0.55$ kN, $P = 8.0$ kN, and $T = 30$ N · m.



Problem 6–14

DESIGN **6–15*** Design the lever arm *CD* of Fig. 6–24 by specifying a suitable size and material.

ANALYSIS **6–16** A spherical pressure vessel is formed of 18-gauge (0.05-in) cold-drawn AISI 1018 sheet steel. If the vessel has a diameter of 8 in, estimate the pressure necessary to initiate yielding. What is the estimated bursting pressure?

*The asterisk indicates a problem that may not have a unique result or a particularly challenging problem.

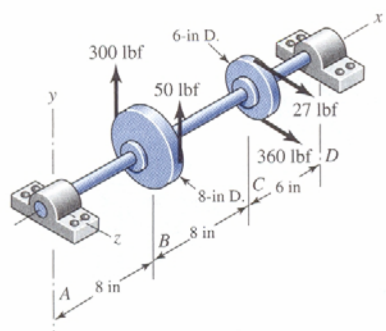
ANALYSIS **6–17** This problem illustrates that the strength of a machine part can sometimes be measured in units other than those of force or moment. For example, the maximum speed that a flywheel can reach without yielding or fracturing is a measure of its strength. In this problem you have a rotating ring made of hot-forged AISI 1020 steel; the ring has a 6-in inside diameter and a 10-in outside diameter and is 1.5 in thick. What speed in revolutions per minute would cause the ring to yield? At what radius would yielding begin? [*Note:* The maximum radial stress occurs at $r = (r_o r_i)^{1/2}$; see Eq. (4–56).]

ANALYSIS **6–18** A light pressure vessel is made of 2024-T3 aluminum alloy tubing with suitable end closures. This cylinder has a $3\frac{1}{2}$-in OD, a 0.065-in wall thickness, and $\nu = 0.334$. The purchase order specifies a minimum yield strength of 46 kpsi. What is the factor of safety if the pressure-release valve is set at 500 psi?

ANALYSIS **6–19** A cold-drawn AISI 1015 steel tube is 300 mm OD by 200 mm ID and is to be subjected to an external pressure caused by a shrink fit. What maximum pressure would cause the material of the tube to yield?

6–20 What speed would cause fracture of the ring of Prob. 6–17 if it were made of grade 30 cast iron?

DESIGN **6–21** The figure shows a shaft mounted in bearings at A and D and having pulleys at B and C. The forces shown acting on the pulley surfaces represent the belt tensions. The shaft is to be made of ASTM grade 25 cast iron using a design factor $n_d = 2.8$. What diameter should be used for the shaft?



Problem 6–21

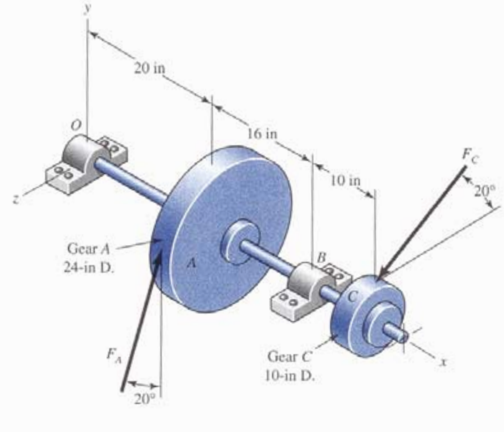
DESIGN **6–22** By modern standards, the shaft design of Prob. 6–21 is poor because it is so long. Suppose it is redesigned by halving the length dimensions. Using the same material and design factor as in Prob. 6–21, find the new shaft diameter.

DESIGN **6–23** The gear forces shown act in planes parallel to the yz plane. The force on gear A is 300 lbf. Consider the bearings at O and B to be simple supports. For a static analysis and a factor of safety of 3.5, use distortion energy to determine the minimum safe diameter of the shaft. Consider the material to have a yield strength of 60 kpsi.

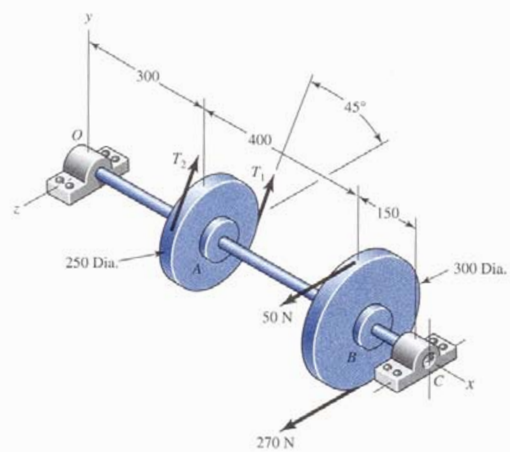
DESIGN **6–24** Repeat Prob. 6–23 using maximum-shear-stress.

DESIGN **6–25** The figure is a schematic drawing of a countershaft that supports two V-belt pulleys. For each pulley, the belt tensions are parallel. For pulley A consider the loose belt tension is 15 percent of the tension on the tight side. A cold-drawn UNS G10180 steel shaft of uniform diameter is to be selected for this application. For a static analysis with a factor of safety of 3.0, determine the minimum preferred size diameter. Use the distortion-energy theory.

Problem 6–23



Problem 6–25
Dimensions in millimeters



DESIGN **6–26** Repeat Prob. 6–25 using maximum shear stress.

ANALYSIS **6–27** The clevis pin shown in the figure is 12 mm in diameter and has the dimensions $a = 12$ mm and $b = 18$ mm. The pin is machined from AISI 1018 hot-rolled steel (Table A–20) and is to be loaded to no more than 4.4 kN. Determine whether or not the assumed loading of figure *c* yields a factor of safety any different from that of figure *d*. Use the maximum-shear-stress theory.

6–28 Repeat Prob. 6–27, but this time use the distortion-energy theory.

ANALYSIS **6–29** A split-ring clamp-type shaft collar is shown in the figure. The collar is 2 in OD by 1 in ID by $\frac{1}{2}$ in wide. The screw is designated as $\frac{1}{4}$-28 UNF. The relation between the screw tightening torque T, the nominal screw diameter d, and the tension in the screw F_i is approximately $T = 0.2\,F_i d$. The shaft is sized to obtain a close running fit. Find the axial holding force F_x of the collar as a function of the coefficient of friction and the screw torque.